An Experimental Method for Measuring Transfer Functions of Acoustic Tubes

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Abstract

This work proposes an experimental method for direct measurement of transfer functions of acoustic tubes. The method obtains a pressure-to-velocity transfer function from measurement of input volume velocity and output pressures of a target tube. Steady sinusoidal waves from 100 Hz to 5 kHz with a 10-Hz increment were used as a source signal. Experimental results compared with transmission line simulations indicate the following: (1) transfer functions obtained from the measurements agree well with those from transmission line simulations; (2) differences between the resonant frequencies obtained from the measurements and simulations with a uniform tube are less than 2.6%. These results show conclusive evidence that the proposed method permits accurate measurements of transfer functions of acoustic tubes.

1. Introduction

This study aims to develop a method for measuring transfer functions of acoustic tubes and evaluate its accuracy. The method obtains a pressure-to-velocity transfer function of tubes with open ends by measuring input volume velocity and output pressures.

Various methods have been proposed to obtain a transfer function of the vocal tract. Fujimura and Lindqvist recorded vocal tract responses by exiting the vocal tract through the laryngeal wall with a vibrator[1]. Sondhi and Resnick measured the input impedance of the vocal tract at the lips and estimated vocal tract area functions[2]. Dang and Honda proposed a method for measuring anti-resonance details of the vocal tract[3]. Besides these acoustical methods, transfer functions have been acquired from morphological data obtained by imaging techniques such as X-ray and magnetic resonance imaging (MRI) by employing transmission line models and the finite element method (FEM).

Our long-term objective is to explore causal factors of speaker characteristics in the vocal tract. To do so, we have made replicas of the vocal tract as shown in Fig. 1 from volumetric MRI data to reproduce individual characteristics of the original speaker by exciting the replica with a sound source[4]. It is hoped that experiments on the replicas help to clarify the effects of the fine structures of the vocal tract on speaker characteristics in detail. The results from such experiments would also be useful as benchmarks for numerical simulations using the transmission line model and FEM.

In this paper, we describe theoretical issues and experimental procedures, and evaluate the accuracy of measurements using cylindrical tubes.

2. Method

2.1. Theoretical considerations

A pressure-to-velocity transfer function of an acoustic tube \( H(\omega) \) is defined as

\[
H(\omega) = \frac{P_{\text{out}}(\omega)}{U_{\text{in}}(\omega)},
\]

where \( \omega \) is the frequency, \( P_{\text{out}}(\omega) \) is the sound pressure at the output end of the tube, and \( U_{\text{in}}(\omega) \) is the volume velocity at the input end of the tube.

A diagram of measuring a transfer function of a target acoustic tube is shown in Fig. 2, where a uniform tube connects a speaker and the target tube. The sound pressures at two adjacent points placed in the uniform tube \( p_1(t) \) and
\( p_2(t) \) are measured to derive \( U_{in}(\omega) \). Assuming plain wave propagation in the uniform tube, the particle velocity \( v_c(t) \) and sound pressure \( p_c(t) \) at the point \( C \) in Fig. 2, i.e., the mid point between the measuring points of \( p_1(t) \) and \( p_2(t) \), are approximately of the form

\[
v_c(t) = -\frac{1}{\rho d} \int_{-\infty}^{t} [p_2(\tau) - p_1(\tau)] d\tau \tag{2}
\]

\[
p_c(t) = \frac{p_1(t) + p_2(t)}{2}, \tag{3}
\]

where \( \rho \) is the air density and \( d \) is a distance between two microphones measuring the sound pressure\[5\]. The microphone distance \( d \) determines the upper valid frequency \( f \) of measurement. For this reason, \( d \) must be

\[
d < \frac{c}{2f}, \tag{4}
\]

where \( c \) is the speed of sound. However, the accuracy of measurement for the lower frequency region deteriorates if \( d \) becomes too small. In this study, therefore, \( d \) is obtained experimentally.

If the length and the area of the section from point \( C \) to the input end of the target tube are given, the sound pressure \( P_{in}(\omega) \) and the particle velocity \( V_{in}(\omega) \) at the input end can be derived by using a transmission matrix of the section

\[
\begin{bmatrix}
P_{in}(\omega) \\
V_{in}(\omega)
\end{bmatrix}
= \begin{bmatrix}
cosh \gamma l & Z \sinh \gamma l \\
Y \sinh \gamma l & \cosh \gamma l
\end{bmatrix}
\begin{bmatrix}
P_c(\omega) \\
V_c(\omega)
\end{bmatrix}
\]

\( \tag{5} \)

Here, \( l \) is the length of the section, \( \gamma \) is the propagation constant based on the length and the area of the section, and \( Z \) and \( Y \) are the characteristic impedance and admittance, respectively\[6\]. In addition, \( P_c(\omega) \) and \( V_c(\omega) \) are \( p_c(t) \) and \( v_c(t) \) on the frequency domain, respectively. Now, the volume velocity \( U_{in}(\omega) \) at the input end of the target tube is

\[
U_{in}(\omega) = \pi r^2 V_{in}(\omega), \tag{6}
\]

where \( r \) is the radius of the uniform tube.

The pressure-to-velocity transfer function of the target tube \( H(\omega) \) is then obtained by Eq. (1) as the ratio of \( U_{in}(\omega) \) derived above and the sound pressure \( P_{out}(\omega) \) measured at the output end of the target tube.

2.2. Experimental approach

Fig. 3 shows a diagram of the measurement setup. An acrylic block was mounted at the radiating end of a horn driver unit (ALE acoustic lab. 7550DE). The acrylic block has a main conduit with a diameter of 5 mm. A source signal is generated by the horn driver unit and propagates in this main conduit to a target tube. A 300-msec sinusoidal wave from 100 Hz to 5 kHz with a 10-Hz increment was used as the source signal. This provides that the frequency resolution for this measurement is 10 Hz. The sound pressure was then measured by three probe microphones (B&K 4182) shown in Fig. 3. Probe microphones M1 and M2 were inserted through narrow holes plumbed perpendicular into the main conduit. The tip of the probe microphones was placed at the mid-line of the main conduit.

The target tube was wrapped in putty (Inaba Denki Sangyo AP-1000-I) to avoid wall vibration, and the tube was glued onto the acrylic block to avert acoustic leaks. A 400 mm × 400 mm plane baffle was attached at the output end of the target tube.

Measurements were carried out in an anechoic room. A room temperature of 25 °C was maintained during the measurements, and the sound velocity \( c \) and the air density \( \rho \) were expected to be 346.37 m/s and 1.1733 kg/m\(^3\) respectively.

The sound pressure was recorded at a sampling rate of 100 kHz with 15-bit resolution. The data obtained by the probe microphones M1 and M2 were completely synchronized. A steady 100-msec section was extracted from the data for each frequency of the sinusoidal wave. A pressure-to-velocity transfer function \( H(\omega) \) was computed after converting the data unit to Pascals by using a pistonphone (B&K 4228).
2.3. Phase characteristics of the probe microphones
The phase characteristics of microphones need to be equal to accurately measure the particle velocity based on Eq. (2). To confirm this condition for the two probe microphones (M1 and M2 in Fig. 3), impulse responses of those probe microphones were measured by using an optimized Aoshima’s time-stretched pulse (OATSP) signal[7]. The experimental results indicate that the two probe microphones have almost equal phase characteristics.

2.4. Transmission line model
To evaluate the accuracy of the measurements, transfer functions were also computed by transmission line models, with consideration given to the effects of viscous and thermal loss. We assumed that wall vibration of target tubes was suppressed in the measurements because the target tubes were wrapped in putty.

The radiation impedance of a target tube \(Z_R\) was approximated by the following equation suggested by Caussé et al.[8]:

\[
\frac{Z_R}{\rho c} = z^2/4 + 0.0127z^4 + 0.082z^4 \ln z - 0.023z^6 + j(0.6133z - 0.036z^3 + 0.034z^3 \ln z - 0.0187z^5),
\]

where \(z\) is the product of \(k\) and \(r\), \(k\) is the wave number, and \(r\) is the radius of the output end. Equation (7) is valid for a frequency region of \(kr < 1.5\).

3. Results

3.1. Transfer function of a uniform tube
A transfer function of a 300-mm aluminum uniform tube was measured. The internal diameter of this uniform tube is 16.8 mm and the wall thickness is 1.6 mm. We set the microphone distance \(d\) to 10 mm, 20 mm, and 30 mm.

The resonant frequencies from the first to the ninth of the uniform tube are listed in Table 1. The resonant frequencies obtained by measurements and the transmission line model were 14 Hz (4.9 %) for \(d = 10\) mm and 4 Hz (1.4 %) for \(d = 20\) mm and 30 mm. These results indicate that the microphone distance \(d\) must be set at 20 mm or 30 mm for the lower-frequency region, while \(d = 10\) mm is sufficient for the other frequency regions. If \(d = 20\) mm is chosen for a frequency region below 500 Hz and \(d = 10\) mm is chosen for that above 500 Hz, the difference in the resonant frequencies obtained between the measurement and the transmission line model are less than 2.6 %.

Figure 4 shows pressure-to-velocity transfer functions obtained from the measurements and transmission line simulations. The microphone distance \(d\) was chosen as described above. These results indicate that this method permits accurate measurement of transfer functions of uniform tubes.

Table 1: The resonant frequencies obtained by measurements and a transmission line model for the 300-mm uniform tube. The microphone distance \(d\) was set to 10 mm, 20 mm, and 30 mm. Values in parentheses are percent differences from resonant frequencies of the transmission line model. The \(n\)th resonant frequency is denoted as \(F_n\).

| model [Hz] | measurement [Hz] |
|---|---|---|
| \(d = 10\) mm | \(d = 20\) mm | \(d = 30\) mm |
| F1 284 | 270 (4.9%) | 280 (1.4%) | 280 (1.4%) |
| F2 859 | 840 (2.2%) | 840 (2.2%) | 840 (2.2%) |
| F3 1,434 | 1,400 (2.4%) | 1,400 (2.4%) | 1,390 (3.1%) |
| F4 2,009 | 1,960 (2.4%) | 1,960 (2.4%) | — |
| F5 2,580 | 2,540 (1.7%) | 2,530 (2.1%) | — |
| F6 3,161 | 3,090 (2.2%) | — | — |
| F7 3,737 | 3,640 (2.6%) | — | — |
| F8 4,313 | 4,220 (2.2%) | — | — |
| F9 4,889 | 4,780 (2.2%) | — | — |

Figure 4: Pressure-to-velocity transfer functions of the 300-mm uniform tube. The microphone distance \(d\) was set at 20 mm for the frequency region below 500 Hz and 10 mm for that above 500 Hz.

3.2. Transfer function of a tube having two diameters
The transfer function of a tube having two sections with different diameters was measured. This tube was actually a concatenation of two 100-mm aluminum tubes; one tube had an internal diameter of 16.8 mm and a wall thickness of 1.6 mm, while another had an internal diameter of 27.0 mm and a wall thickness of 2.0 mm. The diameter of the output end of the tube was 16.8 mm, as shown in Fig. 5. The microphone distance \(d\) was fixed at 10 mm.

The pressure-to-velocity transfer functions of the two
tubes are shown in Fig. 6 and the resonant frequencies from the first to the sixth of the two tubes are listed in Table 2. The difference between the resonant frequencies obtained by the measurement and the transmission line model is less than 4.8%, results which indicate that this method permits accurate measurements of transfer functions of acoustic tubes having a step change in area function.

![Figure 5: A diagram of the tube having two diameters.](image)

![Figure 6: Pressure-to-velocity transfer functions of the tube having two diameters. The microphone distance d is fixed to 10 mm.](image)

### Table 2: The resonant frequencies obtained by measurements and a transmission line model for the tube having two diameters. Values in parentheses are percent difference with resonant frequencies of the transmission line model. The microphone distance d is fixed to 10 mm.

<table>
<thead>
<tr>
<th></th>
<th>model [Hz]</th>
<th>measurement [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>303</td>
<td>290 (4.3%)</td>
</tr>
<tr>
<td>F2</td>
<td>1,418</td>
<td>1,350 (4.8%)</td>
</tr>
<tr>
<td>F3</td>
<td>2,029</td>
<td>1,980 (2.4%)</td>
</tr>
<tr>
<td>F4</td>
<td>3,146</td>
<td>3,010 (4.3%)</td>
</tr>
<tr>
<td>F5</td>
<td>3,758</td>
<td>3,630 (3.4%)</td>
</tr>
<tr>
<td>F6</td>
<td>4,875</td>
<td>4,680 (4.0%)</td>
</tr>
</tbody>
</table>

### 4. Conclusions

This paper described a method for direct measurement of transfer functions of acoustic tubes. This method obtains a pressure-to-velocity transfer function by measuring input volume velocity and output pressures of a target tube. Experimental results supported the feasibility of this method.

This method is applicable not only for cylindrical tubes, but also for bent and asymmetrical tubes. Moreover, the method can also be adopted for an acoustical tube with unknown radiation impedance, such as the vocal tract.

### 5. Acknowledgements

This research was conducted as part of “Research on Human Communication”; with funding from the National Institute of Information and Communications Technology. We wish to thank Dr. Jianwu Dang of JAIST/ATR Human Information Science Laboratories for his helpful comments.

### 6. References


