Two-Mass Models of the Bird Syrinx

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Abstract

Self-sustained vibrations within the syrinx of birds are simulated using symmetric two-mass models. The first system that will be presented here is a rescaled version of the well-known model of human vocal fold vibrations. Moreover, a novel model of the syrinx of a ring dove is introduced. We show that the intensity of harmonics depends strongly on the geometry of the vibrating tissue and the driving pressure. This leads to a discussion how birds can control the strength of overtones.

1 Introduction

Two-mass models of vocal folds vibrations have been used successfully to describe the normal voice [1, 2], vocal fold paralysis [3, 4, 5, 6], phonation onset [7], voice instabilities at high pressures [8] or source-tract coupling [9, 10]. Goller and Larsen [11] and Elemans et al. [12] have shown that sound generation in many birds is induced also by an interaction between air flow and biomechanical vibrations of labia or thickened membranes. Consequently, biomechanical models can be exploited to study the sound generation in the birds syrinx [13, 14, 15].

In this paper we adapt the well-known simplified two-mass model to the dimensions of a syrinx in order to study the onset of sound generation and control of higher harmonics (overtones). In a first model version we simply rescale the two-mass to the size of a songbird syrinx. Then we derive a somewhat more realistic model describing the syrinx of a ring dove. We find that both models exhibit self-sustained oscillations at realistic parameter values. In the classical two-mass model collision occurs at higher pressures leading to strong harmonics. Contrarily, in the model of the dove syrinx collision is avoided leading to more pure tones. Finally, we relate these observations to the widely discussed topic how birds control the intensity of their harmonics [16, 17, 18, 19].

2 Methods and Results

The models discussed in this paper are sketched in Figure 1 and Figure 2 with the parameters listed in the corresponding tables. The first model strongly resembles the widely used simplified two-mass model of vocal folds vibrations [5, 7, 8]. In this model the masses represent the vibrating labia of songbirds according to the findings of Goller and Larsen [11]. The second model is adapted to the syrinx of a ring dove [12]. Here upper and lower masses are joined by plates leading to a much smoother shape of the vibrating tissue. The equations of motions of both models read as follows:

\[
\dot{x}_1 = v_1 \\
\dot{v}_1 = \frac{1}{m_1} \left( F_1 - r_1 v_1 - k_1 x_1 + I_1 - k_c(x_1 - x_2) \right) \\
\dot{x}_2 = v_2 \\
\dot{v}_2 = \frac{1}{m_2} \left( F_2 - r_2 v_2 - k_2 x_2 + I_2 - k_c(x_2 - x_1) \right)
\]

The pressure forces \( F_i \) are derived from the Bernoulli equation and the jet assumption (see [5] for details). The collision forces \( I_i \) are chosen as in

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Figure 1: The rescaled two-mass model of the songbird’s syrinx.

![Diagram of the syrinx model](image1)

Figure 2: The model of the ring dove syrinx: the point masses are joined by three mass-less plates.

![Diagram of the dove syrinx model](image2)

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<thead>
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<th>description</th>
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<td>$l$</td>
<td>length of the glottis</td>
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<td>$a_{01}$</td>
<td>lower rest area</td>
<td>0.0021 cm$^2$</td>
</tr>
<tr>
<td>$a_{02}$</td>
<td>upper rest area</td>
<td>0.00175 cm$^2$</td>
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<td>$d_1$</td>
<td>$1^{st}$ mass thickness</td>
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<td>$d_2$</td>
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<td>$1^{st}$ mass</td>
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<tr>
<td>$m_2$</td>
<td>$2^{nd}$ mass</td>
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<td>$1^{st}$ mass stiffness</td>
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</tr>
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<td>$k_2$</td>
<td>$2^{nd}$ mass stiffness</td>
<td>0.008 g/ms$^2$</td>
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<td>$r$</td>
<td>damping constant ($r_1 = r_2$)</td>
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<tr>
<td>$k_c$</td>
<td>coupling constant</td>
<td>0.0025 g/ms$^2$</td>
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Table 1: Parameters of the rescaled two-mass model shown in Fig. 1.

$W$ | width of the trachea | 0.3 cm |

<table>
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<th>symbol</th>
<th>description</th>
<th>value</th>
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<td>$l$</td>
<td>length of the trachea</td>
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<tr>
<td>$w$</td>
<td>membrane’s width</td>
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<tr>
<td>$d$</td>
<td>$1^{st}$ mass height</td>
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<td>$d + D$</td>
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<td>masses ($m_1 = m_2$)</td>
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</table>

Table 2: Parameters of the model of the ring dove syrinx (see Fig. 2).

...pensate the amount of pressure acting on the first plates on the left and right membranes (see Figure 2). These simulations illustrate that the configuration of the syrinx and the driving pressure can easily control the intensity of harmonics ranging from almost pure tones to quite pronounced overtones.

3 Discussion

It has been shown above that rescaled biomechanical models originally developed to describe vocal fold vibrations can be adapted to model the birds syrinx. Both sound producing organs are excited by the same principle: in the opening phase a high pressure drives the vibrating structure apart and during closure the pressure is reduced due to the Bernoulli force. The fundamental frequency is governed by the mass and stiffness of the vibrating tissue. A slightly convergent rest position leads to
the onset of self-sustained oscillations at realistic pressures (see Figure 1).
Around our default parameters given in Table 1 we found no voice instabilities. Our simulations represent symmetric vibrations. Interestingly, the same model equations can be used to model a single vibrating structure (a “hemi-syrinx”). In this case only the sound intensity is reduced but onset pressure or intensity of harmonics are identical. The amount of harmonics in our simulations was quite variable. In all model version almost pure tones are found near the onset of vibrations (i.e. near the Hopf bifurcation line shown in Figure 3). In the rescaled two-mass model sketched in Figure 1 strong harmonics appear at higher pressures due to collision.

![Figure 3: Variation of the onset pressure depending on the stiffness: at points A and B, i.e. close and relatively far from the Hopf bifurcation, we evaluated the power spectrum of the flow derivative (see Figure 4).](image)

In our model of the ring dove syrinx no collision occurs at default parameters even for very high pressures. Consequently, harmonics are fairly weak. These simulations reveal that the intensities of overtones depend strongly on the configuration of the syrinx and thus muscles can easily control the amount of harmonics.

![Figure 4: Power spectra at two different regimes corresponding to the letters A (solid line) and B (dashed one) in Fig. 3. Close to the Hopf bifurcation point (A) (Ps ≈ 0.034) we observe less intense harmonics. The thick grey line refers to the ring dove model.](image)

There is a long debate about the control of harmonics in bird songs [16, 17, 18, 19, 20]. Often bird songs sound like a whistle and not much energy is found at overtones [18]. However, in some spectrograms very pronounced overtones are visible [11]. In order to imitate human speech [17] or other sounds, birds need a fine control of their overtones. These ongoing discussions can be enlightened by simulations of appropriate models. In a recent paper, Riede et al. illustrated how varying tracheal configurations can suppress the second harmonic in pigeons [16].

![Figure 5: Tracheal area variation during oscillation of the ring dove syrinx model: even with a huge pressure the areas are always greater than zero.](image)

In this paper we have shown that the geometry and the rest position of the syrinx can influence the harmonic spectra drastically. For a small upper mass and a rectangular geometry, collisions leading to strong harmonics can be avoided only near the phonation onset. At higher pressures counteracting forces would be required to diminish collisions. The
avoidance of strong collisions in song birds might be achieved by the medium tympaniform membrane (MTM) attached to the vibrating labia [21]. Such a possible function of the MTM will be discussed in more details in a forthcoming study. In our ring dove model the smoother configuration and equal upper and lower masses counteract collisions even at high pressures. This is presumably due to a stronger effect of the subsyringeal pressure acting on both masses. Recent studies of ring dove vocalizations reveal [22] that there are more harmonics during inspiratory phonation even at low intensities. This implies that asymmetries between outflow and inflow of the air have to be taken into account. This will be treated in a more sophisticated model. Our simulations are a first step towards more realistic modeling of the syrinx. In subsequent studies we will incorporate the MTM and the dynamic control of the associated muscles.

4 Acknowledgment

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References