NUMERICAL SIMULATION OF AIRFLOW THROUGH THE OSCILLATING GLOTTS

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Abstract: The work deals with the numerical solution of 2D unsteady compressible viscous flows in a symmetric channel for a low inlet airflow velocity. The unsteadiness of the flow is caused by a prescribed periodic motion of a part of the channel wall with large amplitudes, nearly closing the channel during the oscillations. The flow in the channel can represent a simplified model of airflow coming from the trachea, through the glottal region with periodically vibrating vocal folds to the human vocal tract.

Keywords: Navier-Stokes equations, unsteady compressible viscous flow, FVM, ALE method, CFD.

I. INTRODUCTION

The fluid-structure interaction problems can be met in many technical and others applications. This study presents the numerical solution of the unsteady compressible viscous flows in a symmetric channel, which is a simplified model of the glottal spaces in the human vocal tract. In reality, the airflow coming from the lungs causes the vocal folds self-oscillations, and the glottis is completely closing in normal phonation regimes generating acoustic pressure fluctuations. In this study, the changes of the channel cross-section are prescribed; the channel is harmonically opening and nearly closing as a first approximation of reality enabling the investigation of the airflow field in the glottal region.

Here, we present the results for the frequency of periodic oscillation 100 Hz and uniform inflow air velocity with the Mach number \( M_a = 0.012 \) at the channel inlet. When the glottis is closing the airflow velocity is becoming much higher in the narrowest part of the airways, where also the viscous forces are important. Therefore for a correct modelling of a real flow in the glottis, the compressible, viscous and unsteady fluid-flow model should be considered.

The authors present the numerical solution and the simulations of the flow field in the human larynx airways performed by the especially developed program.

II. GOVERNING EQUATIONS

Mathematical model: The 2D system of Navier-Stokes equations in conservative non-dimensional form was used as mathematical model to describe the unsteady laminar flow of the compressible viscous fluid in a domain [1]:

\[
W_x + F_x + G_y = \frac{1}{\varepsilon} \left( R_x + S_y \right),
\]

where \( W = [\rho, \rho u, \rho v, e]^T \) is vector of conservative variables, \( F \) and \( G \) are the vectors of inviscid fluxes, \( R \) and \( S \) are the vectors of viscous fluxes, \( Re = (2h \rho f, u_0 \eta) / \eta' \) is Reynolds number given by inflow variables marked by infinity subscript (dimensional variables are marked by the prime), \( \rho \) denotes the density, \( u \) and \( v \) are the components of velocity vector and \( e \) is total energy per unit volume. The static pressure in \( F, G \) is expressed by the equation of state:

\[
p = (\kappa - 1) \left[ e - \frac{1}{2} \rho (u^2 + v^2) \right],
\]

where \( \kappa = 1.4 \) is Poisson constant. The non-dimensional dynamic viscosity in the dissipative terms of equation (1) is the function of temperature: \( \eta = (T/T_C)^{3/4} \).

Mathematical formulation: The computational domain \( D \) is a scale model of channel which shape is inspired by a shape of the vocal folds and supraglottal spaces as shown in Fig. 1. The computational domain is only the lower half of the symmetric channel. The upper boundary is the axis of symmetry, the lower boundary is the channel wall a part of which, between points A and B, is changing the shape according to a given function of time and axial coordinate:

\[
w(x, t) = \begin{cases} a_1 + a_2 \left[ \sin \left( \frac{3\pi}{2} + \pi \frac{x - x_c}{x_n - x_c} \right) + 1 \right] & \text{if } \frac{x - x_c}{x_n - x_c} < 1 \\
2 \left( a_1 + a_2 \right) \cos \left( \frac{\pi}{2} \frac{x - x_n}{x_n - x_c} \right) + 1 & \text{if } \frac{x - x_c}{x_n - x_c} > 1 
\end{cases}
\]

where \( f = 5.83 \times 10^{-3} \) is dimensionless frequency. The gap between the point C and the channel axis is \( g = (d + h) \cdot w(x_C, f) \). The considered dimensions of the domain \( D \) are shown in Tab. 1.

![Fig. 1. Computational domain D.](image)

A simplifying assumption is used that during the normal phonation the vocal folds oscillations are...
symmetric and that the flow in the glottal region is also symmetric.

<table>
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<td>y [-]</td>
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<td>y’ [mm]</td>
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<tr>
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<tr>
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<td>( g_{\max} )</td>
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<td>( L )</td>
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<td>( h )</td>
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### III. NUMERICAL SOLUTION

**Numerical method:** The numerical solution uses finite volume method (FVM) in cell centered form on the grid of quadrilateral cells. Due to the unsteady domain the integral form of FVM is derived using the Arbitrary Lagrangian-Eulerian (ALE) formulation. ALE method defines homeomorphic mapping of reference domain \( D_0 \) at initial time to a domain \( D_t \) at \( t > 0 \) [2].

**Numerical scheme:** The explicit MacCormack (MC) scheme in the predictor (4a) corrector (4b) form in the domain with moving grid of quadrilateral cells is used for the numerical solution of the system (1). The scheme is of the 2nd order of the accuracy in time and space [1]:

\[
W_{i,j}^{n+1/2} = \left[ \frac{D_{i,j}^+}{D_{i,j}^-} \right] W_{i,j}^{n}
\]

\[
- \frac{\Delta t}{\left[ D_{i,j}^+ \right]^2} \sum_{j=1}^{4} \left( \begin{array}{l}
\bar{F}_k - s_n W_k^{n} - \frac{1}{Re} \bar{R}_k \\
\bar{G}_k - s_n W_k^{n} + \frac{1}{Re} \bar{S}_k
\end{array} \right) \Delta y_k
\]  

\[
W_{i,j}^{n+1} = \left[ \frac{D_{i,j}^+}{D_{i,j}^-} \right] \left[ \frac{1}{2} W_{i,j}^{n} + \frac{1}{2} W_{i,j}^{n+1/2} \right]
\]

\[
- \frac{\Delta t}{2 \left[ D_{i,j}^+ \right]^2} \sum_{j=1}^{4} \left( \begin{array}{l}
\bar{F}_k^{n+1/2} - s_n W_k^{n+1/2} - \frac{1}{Re} \bar{R}_k^{n+1/2} \\
\bar{G}_k^{n+1/2} - s_n W_k^{n+1/2} + \frac{1}{Re} \bar{S}_k^{n+1/2}
\end{array} \right) \Delta y_k
\]  

\[
\Delta t \text{ is time step, } \left[ D_{i,j} \right] \text{ is volume of sub-domain } D_{i,j} \text{ in } i,j \text{ position (see Fig. 2) and } \Delta x, \Delta y \text{ are steps of the grid in } x, y \text{ directions. The approximations of the convective terms } sW_k \text{ and the numerical (marked by tilde) viscous fluxes } \bar{R}_k, \bar{S}_k \text{ on edge } k \text{ are central and the vector } s=(s_1, s_2) \text{ represents the speed of the edge } k \text{ (see Fig. 2). The higher partial derivatives of the velocity and the temperature in } \bar{R}_k, \bar{S}_k \text{ are approximated using dual volumes } V_k \text{ (see [1]) as shown in Fig. 2. The inviscid numerical fluxes are approximated by the physical fluxes as follows:}

\[
F_{i,j}^{n+1/2} = F_{i,j}^n, \quad F_{i,j}^{n+1} = F_{i,j}^{n+1/2}, \quad F_{i,j}^{n+1} = F_{i,j}^n, \quad F_{i,j}^{n+1} = F_{i,j}^{n+1/2},
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F_{i,j}^{n+1/2} = F_{i,j}^n, \quad G_{i,j}^{n+1/2} = G_{i,j}^n, \quad G_{i,j}^{n+1/2} = G_{i,j}^n, \quad G_{i,j}^{n+1/2} = G_{i,j}^n, \quad G_{i,j}^{n+1/2} = G_{i,j}^n,
\]

The last term used in MC scheme is the Jameson artificial dissipation term \( AD(W_{i,j}) \) [3, 4]. Then the vector \( \mathbf{W} \) is computed at a new time level \( t^n + \Delta t \):

\[
W_{i,j}^{n+1} = W_{i,j}^{n+1/2} + AD(W_{i,j})
\]

**Grid:** Fig. 3 shows the grid in part of the channel at two time levels (at minimum and maximum of the gap). The minimum cell size in \( y \)- direction is \( \Delta y_{\min} = 1/\sqrt{R_e} \) to resolve capture boundary layer effects (see the detail in Fig. 3, the refinement cells near the wall). The computational domain contains 450x50 cells.

Fig. 2. Finite volume \( D_{ij} \) and the dual volume \( V_k \).

### IV. NUMERICAL RESULTS

The numerical results were obtained for the following input data: Mach number \( M_a = 0.012 \) (\( u^* = 4.1 \text{ m/s} \)), density \( \rho_a = 1.0 \) (\( \rho^* = 1.225 \text{ kg/m}^3 \)), density \( \eta_a = 1/Re \) (\( \eta^* = 1.5 \times 10^{-5} \text{ Pa s} \)), \( Re = 8.5237 \) and atmospheric pressure \( p_a = 1/\kappa \) (\( p^* = 102942 \text{ Pa} \)) at the outlet.

The computation of the unsteady solution was carried out in two stages. Firstly the steady solution is realized, when channel have rigid wall in middle position of the gap \( g = 0.04 \) (0.8 mm). Then the steady solution is used as initial condition for the unsteady simulations.

#### A. The steady solution

Fig. 4(a) shows the steady numerical solution. Results are mapped by iso-lines of Mach number, by streamlines and also by velocity vectors. The maximum of Mach number computed in the domain is \( M_{max} = 0.173 \) at \( x = 2.317 \) on the axis. Fig. 4(b) shows convergence to the steady state solution computed using the L2 norm of momentum residuals (\( \mu \)). The convergence seems to be satisfactory for this very sensitive and complicated case.
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Fig. 3. The grid of the quadrilateral cells in part of the channel at two time levels: at minimum gap \( g_{\text{min}} \) (on top) and at maximum gap \( g_{\text{max}} \) (at the bottom).

\[ s = \frac{(h + d) - w(x, t)}{g} \tag{7} \]

which is the ratio of channel high at the separation point \( x \) and the gap \( g \) at \( x = x_c \).

Fig. 6 shows the detail of the point of flow separation of the instant shown in Fig. 5(b). The flow separation in a narrow divergent channel was predicted in [5, 6] to occur at the point where the glottal-width \( (2g) \) exceeds the minimum glottal width by a fixed amount \((10\% \text{ or } 20\%)\), i.e. for \( s < 1.1; 1.2 \). Our results of the numerical simulations show that the separation parameter can exceed values 8.5 when the gap is close to minimum.

B. The unsteady solution for frequency 100 Hz

The unsteady solution in the fourth period of the wall oscillation is shown in Fig. 5 at several time layers. The highest maximum of Mach number was achieved in instant when the glottal width is opening after the minimum of the gap is exceeded (see Fig. 5(e)) in time \( t = 6\pi + 0.84\pi \ (t' = 0.0342 \text{ s}) \). In this instant the point of flow separation on the wall is \( x = 2.320 \). The points of flow separation depend on the width of the gap \( g \) inversely proportionally (see sub-captions of the Figs. 4 and 5(a)-(f)), where the last numbers denote the separation parameter.

![Fig. 5. The unsteady numerical solution for wall motion - \( f = 100 \text{ Hz}, M_\infty = 0.012, \text{Re} = 5237, \rho_0 = 1/\kappa, 450 \times 50 \text{ cells. Results are mapped by iso-lines of Mach number, by streamlines (lower part of the channel) and by velocity vectors (upper part of the channel).}](image-url)
Fig. 7 shows the changes of the gap \( g \), Mach number and the pressure in real time at the distance \( x=2.3 \) on the channel axis. The phase shifts between the minimum glottal gap \( g \) and the maximum of Mach number and pressure fluctuations are about \( 1.7 \cdot 10^3 \) s and \( 7.8 \cdot 10^4 \) s, respectively. It can be also seen that the flow becomes periodical after the first period of the oscillations.

Fig. 8 shows the Mach number along the axis of symmetry of the channel in several time instants during the oscillation period. Behind the narrowest channel cross-section \( (x=x_c) \) a second peak of the Mach number is forming which travels as a dying wave to the outlet.

V. SUMMARY

The numerical method and the special program code solving the 2D unsteady Navier-Stokes equations for the viscous compressible fluid has been developed. The method has been used for the numerical solution of the airflow in a simplified model of the human vocal tract geometry. Even if no complete closure of the glottis is modeled, the numerical simulation of the airflow field in the glottis is complex and relatively close to reality.

Future tests of the method in modeling of the flow in the human vocal tract will be focused on narrowing the minimum glottal-width \( (2g_{min}<0.02) \), lowering the inlet flow velocity and the geometry of the channel will be closer to a real geometry of the glottis and the vocal tract.

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REFERENCES