

## A NEW DISTANCE MEASURE IN LPC CODING APPLICATION FOR REAL TIME SITUATIONS

Balázs KÖVESI\*, Samir SAOUDI\*, Jean Marc BOUCHER\* and Gábor HORVÁTH†

\* ENST-Br, Dept. SC., Technopôle de Brest Iroise, BP 832, 29285 Brest Cedex, France.

† Technical University of Budapest, Dépt. MMT., Műgyetem rkp. 9. 1521 Budapest, Hungary

### ABSTRACT

The distance measure has a great importance in the phase of the construction of a vector quantizer for LSP parameters as well as in the coding phase. Due to its complexity, the meaningful spectral distance is seldom used for the purpose of quantization. The weighted squared Euclidean distances are mathematically more tractable and are commonly used. Significant differences can be found in the performance of different distances. The aim of this paper is to study different distance measures used in the field of LSP Coding.

A new weighted Euclidean distance will be proposed that not only replaces the spectral distance but estimates well its exact value. However, the use of squared distances will be justified as well. In a real time application, often weights can not be calculated according to the input vector, computation must be done according to the codewords, before coding. This causes some problems in case of split vector quantization or multi stage vector quantization. Some solutions will be given at the end of this paper.

### 1. INTRODUCTION

Line Spectrum Pairs (LSP) or Cosine of LSP (CLSP) provide an efficient representation of the synthesis filter used in the Linear Predictive Coding (LPC) of speech. Vector quantization of these parameters is an important mean of speech compression. In this paper, weighted squared distance measures for vector quantization of LSP (or CLSP) parameters are studied. As relevant articles, we shall use the word distance even if it does not verifies symmetry and/or triangular inequality.

Indeed, a measure of the performance of a quantizer  $Q$  applied to a random vector  $X$  (LSP or CLSP) is given by the expected distance:

$$D(Q) = E\{d(X, Q(X))\} = \int_{RP} d(x, Q(x))p(x) dx \quad (1)$$

where  $p(x)$  is the probability density function of  $X$ . On the other hand, a vector quantizer can be defined by giving the codebook and the distance measure. So the distance measure is highly important in the construction phase of a vector quantizer as well as in the coding phase.

The aim of speech coding is to obtain a synthesized speech signal perceptually as close as possible to the original signal. As the human ear is very sensitive to distortions of the spectrum, spectral distortion has been found to allow the best subjective evaluation of LSP and CLSP encoding quality [3]. During the construction of a vector

quantizer, one needs to find the codeword that minimizes distortion in its class. In case of the spectral distortion, this codeword can not be explicitly computed. During the coding phase, that codeword is chosen as the quantized vector that is the nearest neighbour of the input vector according to the distance measure. Because of its high computational complexity, at present, the spectral distance can not be used in real time applications. That is why the spectral distance is often replaced by a less complex weighted squared Euclidean distance.

### 2. SPECTRAL DISTANCE

The spectral distance of two  $p^{th}$  order LSP vectors  $\vec{\omega}_x$  et  $\vec{\omega}_y$  is defined as follows:

$$d_s(\vec{\omega}_x, \vec{\omega}_y) = \sqrt{\frac{1}{\pi} \int_0^\pi \left\{ 10 \log \frac{S_\omega(\omega, \vec{\omega}_x)}{S_\omega(\omega, \vec{\omega}_y)} \right\}^2 d\omega} \quad [dB] \quad (2)$$

where  $S_\omega(\omega, \vec{\omega})$  is the power spectral density function for the vector  $\vec{\omega} = \{\omega_1, \omega_2, \dots, \omega_p\}$ .

### 3. WEIGHTED DISTANCES FOR REPLACING THE SPECTRAL DISTANCE

In practice, the spectral distance is often replaced by a weighted squared Euclidean distance (3) both in the construction and coding phases.

$$d_{qp}(x, y) = \sum_{i=1}^p w_i |x_i - y_i|^2 \quad (3)$$

The aim is to find a weighting function that gives the best approximation of the spectral distance. There is no simple relationship between the spectral distance and the squared distance. In relevant work, we found many suggestions for the weights, all determined in an empirical way. Here follow some examples of these weights that gave the best results according to our study [3]:

- The weights proposed in [4] are based on the following property of LSP parameters: when a LSP parameter is close to one of its neighbours, the speech spectrum has a peak near that frequency. These LSP parameters have a high spectral sensitivity and should be given higher weights.

$$w_{ai} = \frac{1}{\omega_i - \omega_{i-1}} + \frac{1}{\omega_{i+1} - \omega_i}; \quad i = 1, \dots, p \quad (4)$$

with  $\omega_0 = 0$  and  $\omega_{p+1} = \pi$ . This weighting function is called Inverse Harmonic Mean (IHM), it was used in [6].

- The spectral sensitivity weighting factor with respect to  $\omega_i$ ,  $w_{bi}$  (for example [1]) is defined as:

$$w_{bi} = \int_0^\pi \left| \frac{\partial \log S_\omega(\omega, \vec{\omega})}{\partial \omega_i} \right|^2 d\omega \quad (5)$$

Although the weighted distances mentioned above were defined originally for LSP coefficients, they can be applied for CLSP coefficients as well.

#### 4. A STUDY OF THE RELATIONSHIP BETWEEN THE SPECTRAL AND EUCLIDEAN DISTANCES

The spectral distance is most often replaced by a weighted squared Euclidean distance because of its simplicity. However, nothing justifies the fact that the spectral distance of two LPC vectors is an increasing function of the squared Euclidean distance. That is why, in this section, an experimental study will be described to determine the relationship between the spectral and Euclidean distances.

The following simulations were made to determine the relationship between the spectral distance and the Euclidean distance: a 10<sup>th</sup> order LSP (resp. CLSP) vector was taken and one of its components  $\omega_i$  (resp.  $\zeta_i$ ) was modified by different values to  $\omega_i'$  (resp.  $\zeta_i'$ ). Then, the spectral distance of the original vector  $\vec{\omega} (\omega_1, \dots, \omega_i, \dots, \omega_p)$  and the modified vector  $\vec{\omega}' (\omega_1, \dots, \omega_i', \dots, \omega_p)$  was computed.

The ten vertical dotted lines on figure 1 show the values of the original vector  $\vec{\omega}$ . The ten V-formed solid lines illustrate the results of ten simulations. In each of them, we modified only one of the ten parameters of the vector while the other nine parameters of the two vectors were unchanged. We found that the spectral distance between the original vector and the modified vector is almost in direct proportion with  $|\omega_i' - \omega_i|$ . The same result was obtained in CLSP case as well. So, the spectral distance of the  $\vec{\omega}$  and  $\vec{\omega}'$  vectors can be accurately estimated by the following equation:

$$\hat{d}_s(\vec{\omega}, \vec{\omega}') = \xi_{\omega_i} |\omega_i' - \omega_i| \quad (6)$$

where we called  $\xi_{\omega_i}$  (resp.  $\xi_{\zeta_i}$ ) the spectral distance sensibility of the  $i^{\text{th}}$  LSP (resp. CLSP) parameter. The absolute value of the spectral distance sensibility of a parameter LSP (or CLSP) was found to be independent of the modification sign.

Spectral distance sensitivity can be determined most accurately by linear regression. As the dependency is quasi linear and one point of the straight line is known (when  $\omega_i' = \omega_i$ ),  $\xi_{\omega_i}$  or  $\xi_{\zeta_i}$  can be estimated after computing just one more point of the straight line. We applied this second technique which is less complex than the first one. For example, for computing spectral distance sensitivity  $\xi_{\omega_i}$ , we took  $\omega_i' = \frac{\omega_i - 1 + \omega_i}{2}$  then  $\omega_i'' = \frac{\omega_i + 1 + \omega_i}{2}$  with  $\omega_0 = 0$  and  $\omega_{p+1} = \pi$  ( $\zeta_0 = -2$  and  $\zeta_{p+1} = 2$  in CLSP case). In these two cases, the spectral distances  $d_{s,i,1}$  and  $d_{s,i,2}$  of the original and the perturbed vector were computed. Spectral distance sensitivity  $\xi_{\omega_i}$  was determined as follows:

$$\xi_{\omega_i} = \frac{\frac{d_{s,i,1}}{|\omega_i - \omega_i'|} + \frac{d_{s,i,2}}{|\omega_i'' - \omega_i|}}{2}; \quad i = 1, \dots, p \quad (7)$$

$\xi_{\zeta_i}$  can be obtained in the same way.

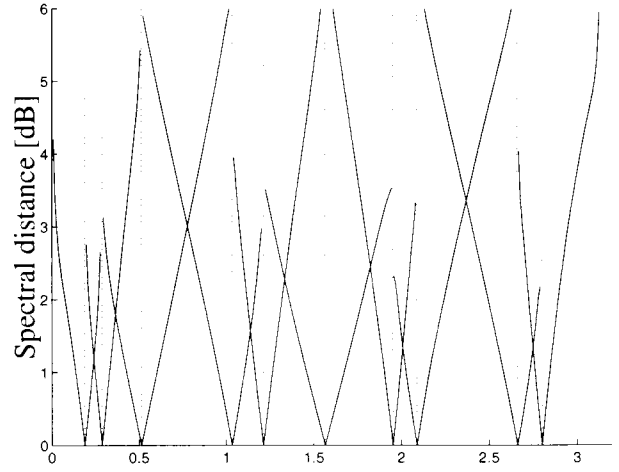


Figure 1: Spectral distance sensitivity of LSP parameters

No.	sensitivity LSP	sensitivity CLSP
1	28.80	82.74
2	23.63	43.06
3	15.99	15.92
4	14.31	9.37
5	17.73	9.55
6	15.64	7.97
7	16.43	8.78
8	16.12	9.83
9	16.53	13.77
10	17.27	22.09

Table 1: Mean spectral distance sensitivities of 10<sup>th</sup> order LSP and CLSP parameters

After computing all spectral distance sensitivities  $\xi_{\omega_i}$ ;  $i = 1, \dots, p$  of an LSP vector  $\vec{\omega}$  (resp.  $\xi_{\zeta_i}$ ;  $i = 1, \dots, p$  of a CLSP vector  $\vec{\zeta}$ ) in this way, we tried to find how the spectral distance of this vector and a completely different vector can be estimated using the sensitivities. We found that the following Euclidean distance gives a good estimation of the spectral distance:

$$\hat{d}_s((\omega_1, \dots, \omega_p), (\omega_1', \dots, \omega_p')) = \sqrt{\sum_{i=1}^p \xi_{\omega_i}^2 (\omega_i' - \omega_i)^2} \quad (8)$$

Using the notation of (3), the proposed weights are equal to the squared spectral distance sensitivities:

$$w_{\zeta_i} = \xi_{\zeta_i}^2 \quad (9)$$

Table 1 gives the mean values of the spectral distance sensitivities of 10<sup>th</sup> order LSP and CLSP parameters estimated on our learning base. The variation of the spectral distance sensitivities of CLSP parameters is greater than that of the LSP parameters. In CLSP case, it is more important to use a weighted distance instead of the squared Euclidean distance. The weights can be defined as the squared mean spectral distance sensitivities. These constant weights will be referred to as  $w_{di}$ .

## 5. COMPARISON OF WEIGHTED EUCLIDEAN AND SQUARED DISTANCES

Usually, the spectral distance is replaced by a weighted squared Euclidean distance (3). In the section above, it was shown that the exact value of the spectral distance can be accurately estimated by a weighted Euclidean (but not squared) distance (8). In this section, the differences between these two distances will be studied.

### 5.1. Research of the nearest neighbour

During quantization, when the nearest neighbour codeword of the input vector is searched, there is no difference between a weighted squared and a weighted non squared distance having the same weights, as the square-root function is strictly an increasing function.

### 5.2. Minimization of the distortion in a class

In the construction phase of a vector quantizer, the codewords  $c_i$  that minimize the given distortion in their class  $s_i$  are periodically searched. Distortion is given by:

$$D(s_i) = \frac{1}{|s_i|} \sum_{x_j \in s_i} d(x_j, c_i) \quad (10)$$

where  $x_j$  is the  $j^{th}$  element of the training set,  $|s_i|$  is the number of the training vectors in the class  $s_i$ . If a squared Euclidean distance (weighted or not) (3) is used, it is easy to see that the codeword that minimizes distortion (10) is simply the gravity center of the class:

$$c_i = \frac{1}{|s_i|} \sum_{x_j \in s_i} x_j \quad (11)$$

This property is the big advantage of the squared Euclidean distance, and explains its frequent use. Using a weighted Euclidean distance (8), by canceling the partial derivatives of distance, the following equation is obtained:

$$c_i = \frac{\sum_{x_j \in s_i} \frac{x_j}{d_{ep}(x_j, c_i)}}{\sum_{x_j \in s_i} \frac{1}{d_{ep}(x_j, c_i)}} \quad (12)$$

The searched codeword can not be explicitly expressed. The equation (12) suggests an iterative solution to compute the codeword that minimizes distortion in the class  $s_i$ . However, in case of real LSP and CLSP vectors, it was found that the gravity center of the class  $s_i$  and the codeword that minimizes the Euclidean distortion (obtained by the iterative method) are very close to each other. The reason is the nearly symmetric distribution of the vectors in the class. Thus, it was found that the use of the corresponding squared weighted Euclidean distance (3) to replace the spectral distance is justified, although the approximation with the simple weighted Euclidean distance is more precise.

## 6. USING SQUARED WEIGHTED EUCLIDEAN DISTANCES IN THE CODING PHASE

In vector quantization, the distance of an input vector and a codeword is computed. The weights depend on one of the two vectors, there are three main possibilities to determine them:

Distance	Spectral distortion [dB]
Squared Euclidean (SE)	2.84
Weighted SE: $w_{ai}$	2.77
Weighted SE: $w_{bi}$	2.74
Weighted SE: $w_{ci}$	2.74
Weighted SE: $w_{di}$	2.82
Spectral	2.72

Table 2: Spectral distortion of coding LSP vectors in function of the distance measure.

- The calculation of the weights can be based on the input vector. This solution was used in each paper mentioned above. Its disadvantage is that the complexity of the coding increases seriously. This solution has no sense especially in case of the weights  $w_{bi}$  and  $w_{ci}$ . It is less complex to calculate the spectral distance directly than to determine these weights for each input vector.
- Here, we suggest determining the weights according to the codewords. As the codebook is fixed during the coding phase, the calculations can be made before quantization and the weights can be stored in memory. This solution double the memory requirements of the codebook but it is more suitable for real time applications.
- To use the constant weights like  $w_{di}$ , the mean value of the weights estimated on a learning set can be taken. Of course, this solution is less precise, but the increase of both computational time and memory needed is negligible.

## 7. EXPERIMENTAL RESULTS

To compare the different distance measures, first, a  $C_q$  codebook of 1024 codewords was constructed by the k-means algorithm using a training set of 150000 vectors. In the construction phase, the squared Euclidean distance was used without weights. We then quantized 35000 testing vectors, separated from the training set, by this codebook using different distance measures. The individual weights were determined according to the codewords. Table 2 gives the spectral distortion obtained in function of the used distance in case of the LSP vectors.

The distance weighted by the spectral sensitivity ( $w_{bi}$ ) and by our proposed squared spectral distance sensitivity ( $w_{ci}$ ) is quite near to the performance of the spectral distance. The use of the weighted Euclidean distance (8) by the squared spectral distance sensitivities  $w_{ci}$ , estimates the value of the spectral distortion well. For example, in LSP case, the estimated value was 2.70 dB while the exact value was 2.74 dB.

The weighted squared Euclidean distances can be used in the learning and the coding phase as well. The following experience shows that it is more important to use a correct distance measure in the coding phase. For the construction of the  $C_p$  codebook of 1024 CLSP vectors we used the weighted, by the squared spectral distance sensitivities, squared Euclidean distance. In table 3, four different cases can be compared.

Codebook and distance	Spectral distortion [dB]
$C_q$ + Squared Euclidean (SE)	3.06
$C_q$ + Weighted SE; $w_g i$	2.85
$C_p$ + Squared Euclidean (SE)	3.15
$C_p$ + Weighted SE; $w_g i$	2.77

Table 3: Spectral distortion of coding CLSP vectors in function of the distance measure and the codebook.

## 8. APPLICATION FOR REAL TIME SITUATIONS

To be able to implant a vector quantizer in a real time application, often a sub-optimal solution has to be used as the Multi Stage Vector Quantization (MSVQ) [2] or the Split Vector Quantization (SVQ) [5]. In this section, these solutions will be studied from the point of view of the used distance. It will be supposed that the individual weights can not be calculated according to the input vector because of the time constraint of a real time application, they have to be determined according to the codewords of the codebooks and stored in memory before coding. The problem is that all individual weights ( $w_{a_i}$ ,  $w_{b_i}$ ,  $w_{c_i}$ ) are based on special properties of LSP (or CLSP) vectors and the knowledge of the whole vector is needed to calculate them.

### 8.1. Multi Stage Vector Quantization

In case of the MSVQ, the codewords of the second and further stages represent the quantization error of the previous stages. These vectors do not have the same properties as the LPC vectors, the individual weights can not be determined according to them. As the codeword chosen in the first stage is near to the codeword corrected by the further stages, we suggest using the weights of the codeword chosen on the first stage.

This approximation is better if the quantization on the first stage is more precise. That is why it is important to assign as many bits as possible to the first stage.

### 8.2. Split Vector Quantization

In case of SVQ, the codebooks contain only a part of the codewords that not allow the computation of individual weights. Our proposition is based on the following properties of LSP and CLSP parameters:

- The spectral influence of an LSP (or CLSP) parameter is localized in its region.
- The closeness of two LSP parameters marks a spectrally important region.

That is why we completed the sub-vectors in the codebooks by spectrally neutral fictitious sub-vectors and we determined the weights on this completed vector. For example, in case of  $10^{th}$  order LSP vectors, if a codeword contains the last four coefficients  $\omega_7, \dots, \omega_{10}$ , we defined the six fictitious coefficients like  $\omega_i = \omega_{min} + i \frac{\omega_7 - \omega_{min}}{7}$ ,  $i = 1, \dots, 6$ , with  $\omega_{min} = 0$ . The uniform distribution of fictitious LSP parameters assure the spectral neutrality of this part of the vector.

In the case of CLSP parameters, we generated the fictitious CLSP parameters in the LSP field to assure spectral neutrality, as the mean LSP vectors are more uniformly distributed. For example, if a codeword contains

the first six coefficients  $\zeta_1, \dots, \zeta_6$ , we calculated the rest four coefficients as

$$\zeta_i = -2 \cos \left( \arccos(-\zeta_6/2) + (i-6) \frac{\omega_{max} - \arccos(-\zeta_6/2)}{5} \right),$$

$$i = 7, \dots, 10, \text{ with } \omega_{max} = \pi.$$

Of course, constant mean weights like  $w_{d_i}$  can be used in both cases without difficulties. But the solutions proposed above give less spectral distortion.

## 9. CONCLUSIONS

In this paper we studied and compared the weighted squared Euclidean distances that can replace the distance spectral. A new weighted Euclidean distance was proposed that was found to be the best approximation of the spectral distance. However, by using, instead, the corresponding squared distance that is mathematically less complex, the error is negligible. We showed how individual weights can be determined according to the codewords of the codebook in case of split vector quantization or multi stage vector quantization. By storing the obtained weights in memory, the calculation complexity is less important which is often essential in a real time application.

## 10. REFERENCES

- [1] S. BRUHN. Efficient interblock noiseless coding of speech LPC parameters. In *Proc. ICASSP-94, IEEE International Conference of Acoustics, Speech, and Signal Processing*, volume 1, pages 1.501–1.504, Adelaide, Australia, April 1994.
- [2] B.H. JUANG and A.H. GRAY. Multiple stage vector quantization for speech coding. In *Proc. ICASSP-82, IEEE International Conference of Acoustics, Speech, and Signal Processing*, volume 1, pages 597–600, Paris, France, May 1982. IEEE.
- [3] B. KÖVESI. *Quantification vectorielle des paires de raies spectrales pour la compression de la parole à débit réduit. Application à la téléphonie numérique sur canal acoustique sous-marin*. Thèse de doctorat, Université de Rennes I, Janvier 1997.
- [4] R. LAROIA, N. PHAMDO, and N. FARVARDIN. Robust and efficient quantization of speech LSP parameters using structured vector quantizers. In *Proc. ICASSP-91, IEEE International Conference of Acoustics, Speech, and Signal Processing*, volume 1, pages 641–644, Toronto, Canada, May 1991.
- [5] E. PAKSOY, W.Y. CHAN, and A. GERSHO. Vector quantization of speech LSP parameters with generalized product codes. In *Proc. ICLSP-92, International Conference on Spoken Language Processing*, volume 1, pages 33–36, Banff, Alberta, Canada, October 1992.
- [6] J. PAN and T.R. FISCHER. Vector quantization - lattice quantization of speech LPC coefficients. In *Proc. ICASSP-94, IEEE International Conference of Acoustics, Speech, and Signal Processing*, volume 1, pages 1.513–1.516, Adelaide, Australia, April 1994.